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Nonperturbative corrections of quark and gluon condensates to QCD inspired quark potentials

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Abstract. We adopt the Lorentz gauge to derive the non-local two-gluon vacuum expectation value (VEV) with translational invariance. By means of the obtained non-local two-gluon VEV, the leading nonperturbative QCD corrections to one gluon exchange quark-quark, quark-antiquark and $q\bar{q}$ pair-excitation potentials are given by employing non-vanishing vacuum condensates of quarks and gluons to modify the free gluon propagator. The linear, cubic and Yukawa-type terms in quark-quark potential appear automatically. In the $q\bar{q}$ pair-excitation potential with $\omega_q = 0$, the linear, square and cubic terms arise from the nonzero quark and gluon condensates.

1 Introduction

Recently, a great progress has been made in our understanding of strong interaction in the framework of quantum chromodynamics(QCD): the quark potential model based on one-gluon exchange approximation can reproduce the baryon spectrum and the static properties of hadrons [1] correctly, especially the positive parity states; including the $q\bar{q}$ creation and annihilation terms in the one-gluon-exchange approximation, it can give a description of meson-nucleon interactions [2]. It is well understood that the one-gluon exchange can generate only the short-range part of the baryon-baryon interaction since the one-gluon exchange potential is the nonrelativistic reduction of the operator derived in the perturbative QCD scheme. It is clear that, to reflect medium- and long-range QCD, some nonperturbative effects induced by the complicated structure of QCD vacuum should be taken into account. This kind of nonperturbative effect on the quark interaction has become one of the interesting topics [3–5] recently. It was firstly suggested in QCD sum rules [6] that these nonperturbative effects can be phenomenologically considered by introducing nonvanishing condensates of quarks and gluons into the Green function of QCD by means of the operator-product expansion(OPE) [7]. They started from short distance, where the quark-gluon dynamics is essentially perturbative, and extrapolated the dynamics to larger distance by introducing non-perturbative effects step by step [8]. The application of the theory in studying hadronic properties indicates that one can trust the validity of this approach. Inspired by the success of the QCD sum rules, we introduced the non-perturbative QCD effects into the traditional potential model [9] and investigated the possible effects of quark and gluon condensates in heavy quarkonium spectra [10]. A deeper understanding of the hadronic structure and a underlying mechanism which determines how quarks are bound into hadrons have been obtained.

In the above non-perturbative consideration, the non-local two-quark and two-gluon vacuum expectation values (VEVs) are essential in deriving an effective potential. To evaluate the non-local two-gluon VEV, one usually adopted the fixed-point gauge of the vacuum gluon field [11]. However, the fixed-point gauge violates translational invariance of the non-local two-gluon VEV. Such a problem becomes especially prominent in the evaluation of the gluon condensate contributions to the three-point non-perturbative vertex [12]. Furthermore, there exist differences in the result of Shen et al. [9] and that of Larsson [13] in the calculation of the condensate modified gluon propagator which is included in a proper scattering amplitude for making the unitary expansion. The result of Shen et al. [9] was obtained in the standard way, in which the normal product operators such as $q\bar{q}(0)$ and $G^2(0)$ have non-vanishing matrix elements in the physical vacuum, — i.e., $\langle 0|q\bar{q}|0\rangle$ and $\langle 0|G^2|0\rangle$ are left as parameters to describe non-perturbative effects. The modified gluon propagator in momentum space can be written as [9]

$$G_{\mu\nu} = \frac{-i}{q^2}(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})F(q^2) \tag{1}$$

where

$$F(q^2) = 1 + \frac{1}{3}g^2 \sum_{\beta=u,d,s} \frac{m_\beta \langle 0|q_\beta \bar{q}_\beta|0\rangle}{q^2(q^2 - m_\beta^2)}$$

$$+\frac{9}{32}g^2\langle 0|G^2|0\rangle\frac{1}{q^4}. \quad (2)$$

On the other hand, Larsson's result [13] reads

$$D_{\mu\nu} = \left[1 - \sum_\beta \frac{g^2 m_\beta\langle 0|q_\beta\bar{q}_\beta|0\rangle}{q^2(q^2+m_\beta^2)} + \frac{5g^2\langle 0|G^2|0\rangle}{288q^4}\right]^{-1} \times\frac{(-i)}{q^2}(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}), \quad (3)$$

which was obtained by comparing the $(2n+1)$-point Green's function (n is the number of external legs of q or B^μ) and the n-point Green's function with the insertion of the operators $q\bar{q}(0)$ or $G^2(0)$ (these Green's functions are with respect to the physical vacuum). Expanding the first piece in (3) with respect to α_s, one can find that not only the coefficients at the lowest order of α_s are different from those in (2), but also the sign of the coefficient associated with the gluon condensate is different from that in (2).

The discrepancy between (2) by Shen and (3) by Larsson comes from the difference in the gauge conditions that they worked with. When all the gauge invariant set of diagrams (in this case, gluon propagator, quark-gluon vertex and quark propagator corrections) are summed up, the result is unique and should be independent of the gauge conditions.

Deriving the condensate corrections to QCD inspired quark potentials in Lorentz gauge with translational invariance is the main purpose of this paper. We first derive the non-local two-gluon VEV with translational invariance in Lorentz gauge. Then, using the obtained two-gluon VEV, we extend the leading nonperturbative QCD corrections to the perturbative one gluon exchange quark-quark potential in [9], where the fixed-point gauge of the vacuum gluon field was adopted, to include the $q\bar{q}$ pair-annihilation and $q\bar{q}$ pair-excitation potentials by considering the contributions of the quark and gluon condensates $\langle\bar{q}q\rangle$ and $\langle GG\rangle$ to the gluon propagator. In addition, to get a better description of corrections due to the quark condensate, we keep the terms up to the next-to-leading-order in the full coefficient of $\langle\bar{q}q\rangle$ component of the nonperturbative two-quark VEV [14], whereas only leading-order term was used in [9].

In detail, we organize the paper as follows: after deriving the non-local two-gluon VEV with translational invariance in Lorentz gauge, we evaluate in Sect. 3 the nonperturbative corrections to quark-quark potential by using the two-gluon and two-quark VEV with translational invariance. In Sects. 4 and 5, we extend the nonperturbative calculation to the one-gluon exchange potentials of the $q\bar{q}$ pair-annihilation and $q\bar{q}$ pair-excitation, respectively. In the final section, we give a brief discussion and conclusion with a few remarks on possible extensions of present work.

2 Non-local two-gluon VEV with translational invariance

In this section, we derive the non-local two-gluon VEV with translational invariance in Lorentz gauge.

In the fixed-point gauge,

$$x_\mu B_a^\mu(x) = 0, \quad (4)$$

the non-local two-gluon VEV is [6,14]

$$\begin{aligned}\langle 0|B_\mu^a(x)B_\nu^b(y)|0\rangle &= \frac{1}{4}x^\rho y^\sigma\langle 0|G_{\rho\mu}^a G_{\sigma\nu}^b|0\rangle + \cdots\cdots \\ &= \frac{\delta_{ab}}{48(N_c^2-1)}x^\rho y^\sigma(g_{\rho\sigma}g_{\mu\nu} - g_{\rho\nu}g_{\sigma\mu}) \\ &\quad\times\langle 0|G^2|0\rangle + \cdots, \end{aligned} \quad (5)$$

where

$$\langle 0|G^2|0\rangle = \langle 0|G_{\rho\mu}^a G_a^{\rho\mu}|0\rangle. \quad (6)$$

The expansion (5) violates the translational invariance since the right hand side(RHS) of (5) is a function of xy instead of $(x-y)$.

In order to obtain the expansion of $\langle 0|B_\mu^a(x)B_\nu^b(y)|0\rangle$ with translational invariance, we study the basic requirements for translational invariance. Assume that $f(x)$ and $g(x)$ are arbitrary composite fields, the translational invariance means

$$\begin{aligned}\langle 0|f(x)g(y)|0\rangle &= \langle 0|f(x-y)g(0)|0\rangle \\ &= \langle 0|f(0)g(y-x)|0\rangle\end{aligned} \quad (7)$$

from which we get

$$\langle 0|\partial_\rho f(0)g(0)|0\rangle = -\langle 0|f(0)\partial_\rho g(0)|0\rangle. \quad (8)$$

For example, when f and g are gluon fields, then

$$\langle 0|\partial_\rho B_\mu^a(0)B_\nu^b(0)|0\rangle = -\langle 0|B_\mu^a(0)\partial_\rho B_\nu^b(0)|0\rangle. \quad (9)$$

On the other hand, according to the translational invariance of $\langle 0|B_\mu^a(x)B_\nu^b(y)|0\rangle$, we get

$$\begin{aligned}\langle 0|B_\mu^a(x)B_\nu^b(y)|0\rangle &= \pi_{\mu\nu}^{ab}(x-y) \\ &= \int \Pi_{\mu\nu}^{ab}(k)\mathrm{e}^{-\mathrm{i}(\mathrm{x}-\mathrm{y})\cdot\mathrm{k}}\frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4}\end{aligned} \quad (10)$$

with

$$\Pi_{\mu\nu}^{ab}(k) = \pi(k^2)(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\delta_{ab} \quad (11)$$

which follows from the Lorentz gauge condition

$$\partial^\mu B_\mu^a(x) = 0. \quad (12)$$

Expanding the exponential term in (10), in consideration of the fact that the terms with the odd powers of k vanish after integrating over k, we have

$$\begin{aligned}\langle 0|B_\mu^a(x)B_\nu^b(y)|0\rangle &= \int \pi(k^2)(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\delta_{ab} \\ &\quad\times\left[1 + \frac{1}{2!}(-i(x-y)\cdot k)^2 + \cdots \right. \\ &\quad\left. + \frac{1}{(2n)!}(-i(x-y)\cdot k)^{2n} + \cdots\right] \\ &\quad\times\frac{d^4k}{(2\pi)^4}\end{aligned} \quad (13)$$

from which we obtain

$$\langle 0|\partial_\rho B^a_\mu(0)B^b_\nu(0)|0\rangle = 0 \tag{14}$$

and

$$\langle 0|\partial_\rho\partial_\sigma\partial_\lambda B^a_\mu(0)B^b_\nu(0)|0\rangle = 0 \tag{15}$$

and so on. Upon contraction of both sides of (15) with $g^{\rho\sigma}$ we have

$$\langle 0|\partial^2\partial_\lambda B^a_\mu(0)B^b_\nu(0)|0\rangle = 0. \tag{16}$$

Equations (14)–(16) embody the requirements for the translational invariance of non-local two-gluon VEV. According to these requirements, the non-local two-gluon VEV can be expressed as

$$\begin{aligned}\langle 0|B^a_\mu(x)B^b_\nu(y)|0\rangle &= \langle 0|B^a_\mu(0)B^b_\nu(0)|0\rangle \\ &-\frac{\delta_{ab}}{2(N_c^2-1)}(x-y)^\rho(x-y)^\sigma \\ &\times\langle 0|\partial_\rho B^d_\mu(0)\partial_\sigma B^d_\nu(0)|0\rangle+\cdots\cdots,\end{aligned} \tag{17}$$

where

$$\begin{aligned}\langle 0|B^a_\mu(0)B^b_\nu(0)|0\rangle &= \frac{g_{\mu\nu}}{4}\frac{\delta_{ab}}{(N_c^2-1)}\langle 0|B^d_\rho(0)B^\rho_d(0)|0\rangle \\ &= \frac{g_{\mu\nu}}{4}\frac{\delta_{ab}}{(N_c^2-1)}\langle 0|B^2|0\rangle\end{aligned} \tag{18}$$

and

$$\begin{aligned}&\frac{1}{2}\langle 0|\partial_\rho B^a_\mu(0)\partial_\sigma B^a_\nu(0)|0\rangle \\ &=\left[Sg_{\mu\nu}g_{\rho\sigma}+\frac{R}{2}(g_{\rho\nu}g_{\sigma\mu}+g_{\rho\mu}g_{\nu\sigma})\right].\end{aligned} \tag{19}$$

Contracting (19) with $g^{\rho\sigma}g^{\mu\nu}$ and $g^{\rho\mu}g^{\sigma\nu}$ leads to

$$\frac{1}{2}\langle 0|\partial^\sigma B^\nu_a(0)\partial_\sigma B^a_\nu(0)|0\rangle = 16S+4R \tag{20}$$

and

$$\frac{1}{2}\langle 0|\partial^\mu B^a_\mu(0)\partial^\nu B^a_\nu(0)|0\rangle = 4S+10R. \tag{21}$$

respectively. According to the Lorentz gauge condition (12), (21) means

$$R=-\frac{2}{5}S. \tag{22}$$

Furthermore, using the definition of the gluon field strength and only keeping the contribution of the vacuum intermediate state [15], one can easily find that

$$\begin{aligned}&\langle 0|G^a_{\rho\mu}(0)G^b_{\sigma\nu}(0)|0\rangle \\ &=\left[\langle 0|\partial_\rho B^a_\mu(0)\partial_\sigma B^b_\nu(0)|0\rangle+\langle 0|\partial_\mu B^a_\rho(0)\partial_\nu B^b_\sigma(0)|0\rangle\right] \\ &-\left[\langle 0|\partial_\mu B^a_\rho(0)\partial_\sigma B^b_\nu(0)|0\rangle+\langle 0|\partial_\rho B^a_\mu(0)\partial_\nu B^b_\sigma(0)|0\rangle\right] \\ &+\frac{g^2N_c}{12(N_c^2-1)^2}\delta_{ab}\left[g_{\rho\sigma}g_{\mu\nu}-g_{\mu\sigma}g_{\rho\nu}\right]\langle 0|B^2|0\rangle^2\end{aligned} \tag{23}$$

which results in

$$S=\frac{5\langle 0|G^2|0\rangle}{288}-\frac{5N_cg^2}{288(N_c^2-1)}\langle 0|B^2|0\rangle^2. \tag{24}$$

By considering (22), (17) can also be rewritten as

$$\begin{aligned}\langle 0|B^a_\mu(x)B^b_\nu(y)|0\rangle &= \frac{\delta_{ab}}{(N_c^2-1)}\frac{g_{\mu\nu}}{4}\langle 0|B^2|0\rangle \\ &-\frac{\delta_{ab}}{(N_c^2-1)}S\left[(x-y)^2g_{\mu\nu}\right. \\ &\left.-\frac{2}{5}(x-y)_\mu(x-y)_\nu\right]+\cdots\cdots\end{aligned} \tag{25}$$

with the translational invariance. Note that, on the RHS of (25), beyond the second term, which has already been used by Bagan et al. [16] with neglecting $\langle 0|B^2|0\rangle^2$, we also have the first term $\langle 0|B^2|0\rangle$ which should not be omitted according to the value given in [15] or our following estimate of its value. The anti-Fourier transformation for the RHS of (25) reads

$$\begin{aligned}C(k) &= \frac{\delta_{ab}g_{\mu\nu}\langle 0|B^2|0\rangle}{4(N_c^2-1)}\delta^4(k)+\frac{\delta_{ab}S}{(N_c^2-1)}\left[g_{\mu\nu}g_{\rho\sigma}\right. \\ &\left.-\frac{2}{5}g_{\mu\rho}g_{\nu\sigma}\right]\frac{\partial^2}{\partial k_\rho\partial k_\sigma}\delta^4(k)+\cdots\cdots.\end{aligned} \tag{26}$$

In order to obtain a workable formula, the value of $\langle 0|B^2|0\rangle$ needs to be estimated. According to the scheme suggested in [17], we can determine the value of $\langle 0|B^2|0\rangle$ with the phenomenological values for the vacuum condensates $\langle 0|G^2|0\rangle$ and $\langle 0|q\bar{q}|0\rangle$ through the QCD equation of motion

$$D^{ab}_\mu(B)G^{\nu\mu}_b(B)=g\bar{q}\gamma^\nu t^a q \tag{27}$$

with

$$D^{ab}_\mu(B)=\delta_{ab}\partial_\mu-gf^{abc}B^c_\mu. \tag{28}$$

Multiplying (27) by itself and taking VEV, we have

$$\begin{aligned}&\langle 0|D^{ac_1}_\rho(B(x))G^{\mu\rho}_{c_1}(B(x))D^{bc_2}_\lambda(B(x))G^{\nu\lambda}_{c_2}(B(x))|0\rangle \\ &=g^2\langle 0|\bar{q}(x)\gamma^\mu t^a q(x)\bar{q}(x)\gamma^\nu t^b q(x)|0\rangle.\end{aligned} \tag{29}$$

By means of (14)-(16), expanding the left hand side(LHS) of the above equation and keeping the contribution of the vacuum intermediate state [15], we obtain

$$\begin{aligned}&\langle 0|B^2|0\rangle^3-\frac{11(N_c^2-1)}{36N_c\alpha_s}\langle 0|G^2|0\rangle\langle 0|B^2|0\rangle \\ &=\frac{(N_c^2-1)^3\langle 0|q\bar{q}|0\rangle^2}{9\pi N_c^4\alpha_s},\end{aligned} \tag{30}$$

where

$$\langle 0|q\bar{q}|0\rangle=\sum_f\langle 0|q_f\bar{q}_f|0\rangle, \tag{31}$$

The phenomenological vacuum expectation values of $\langle 0|q_f\bar{q}_f|0\rangle$ and $\langle 0|G^2|0\rangle$ are given as [14,17]

$$\begin{aligned}\langle 0|u\bar{u}|0\rangle &= \langle 0|d\bar{d}|0\rangle \\ &= 1.3\langle 0|s\bar{s}|0\rangle=-(250MeV)^3,\end{aligned} \tag{32}$$

and

$$\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle=(360MeV)^4. \tag{33}$$

The effective coupling constant $\alpha_s = g^2/4\pi$ in (30) is adopted as $\alpha_s = 0.5$. It is noteworthy that (30) is only valid under the Lorentz gauge condition. Although the "pairing" order parameter $\langle 0|B^2|0\rangle$ is clearly not gauge invariant, its specified value in the Lorentz gauge can be determined according to (30). Therefore, the physically observable quantities in the following sections can be expressed via this specified value of $\langle 0|B^2|0\rangle$, which is equivalent to that all descriptions are made in Lorentz gauge. There are three real roots of $\langle 0|B^2|0\rangle$ in (30). The numerical calculation shows that the magnitude of two roots decreases as α_s, and the third root increases as α_s. Physically, increase of α_s means the strengthening of nonperturbative effect, thus only the third root is reasonable. When $\alpha_s = 0.5$, we find

$$\langle 0|B^2|0\rangle = -(127MeV)^2 \quad (34)$$

which should not be omitted in (24)-(26). Equation (24) yields

$$S = (206MeV)^4. \quad (35)$$

Thus, we obtain a workable formula of the non-local two-gluon VEV as shown in (26) with $\langle 0|B^2|0\rangle$ of (34) and S of (35).

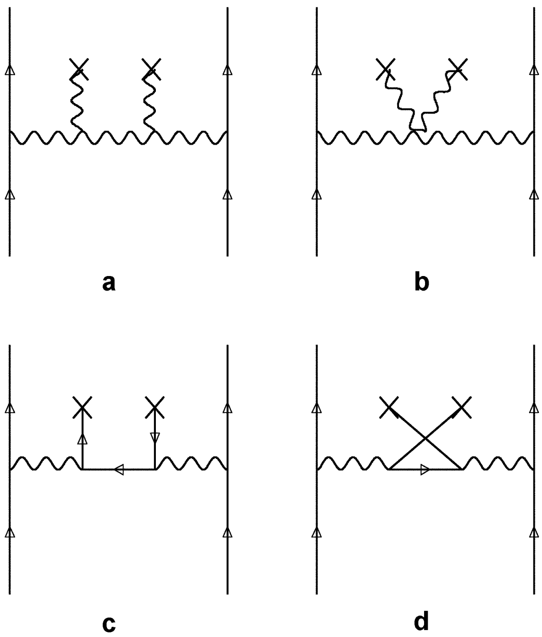

Fig. 1a–d. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative quark-quark potential in one-gluon exchange approximation with the lowest dimensional quark and gluon condensates

3 Nonperturbative QCD corrections to the quark-quark potential

Firstly, we briefly introduce the nonrelativistic reduction method which is used to extract nonperturbative corrections to perturbative QCD potentials. According to [18], the reduction formula for the connected part of the S-matrix element can be expressed as:

$$\langle p_1', p_2'\ out|p_1, p_2\ in\rangle_c = \psi_i\psi_f(2\pi)^4\delta^4(p_1 + p_2 - p_1' - p_2') \times G_{\rm trunc}(-p_1', -p_2'; p_1, p_2)|_{p_1^2={p_1'}^2=m_1^2; p_2^2={p_2'}^2=m_2^2}, \quad (36)$$

where ψ_i and ψ_f are the wave functions of the initial and final states respectively and $G_{\rm trunc}$ is a truncated Green function which contains the VEV of some local product of quark and gluon fields. It is impossible in QCD to compute the Green function exactly because of the complex structure of the QCD vacuum, thus one has to introduce some approximation. Generally, Wilson's OPE method [7], where nonzero vacuum matrix elements of composite operators are used as a parameterization of long-distance nonperturbative effects, is employed to extract the perturbative part of the S-matrix element as well as its nonperturbative part arising from the nonzero quark and gluon condensates. Therefore, we can use the method of unitary expansion of the scattering operator and then make the nonrelativistic reduction to obtain nonperturbative corrections to the perturbative QCD potential.

The Feynman diagrams that will be used to evaluate the nonperturbative correction to quark-quark interaction are shown in Fig. 1. For the scattering of the two quarks of different flavors, by using the obtained two-gluon VEV of (26), the contribution of the Feynman diagram 1(a) to the S-matrix is

$$\begin{aligned} S_{1(a)}(p_1, p_2; p_1', p_2') &= ig^4[\bar{\psi}^-(p_2')\gamma^\mu \frac{\lambda^a}{2}\psi^+(p_2)] \\ &\times[\bar{\psi}^-(p_1')\gamma^\nu \frac{\lambda^b}{2}\psi^+(p_1)]D^{aa'}_{\mu\mu'}(q)\int d^4k \left[\delta^4(k)\frac{\langle 0|B^2|0|\rangle g_{\rho\sigma}}{4(N_c^2-1)}\right. \\ &\left.+\frac{S}{N_c^2-1}\left(g_{\rho\sigma}g_{lm} - \frac{2}{5}g_{\rho l}g_{\sigma m}\right)\frac{\partial^2}{\partial k_l \partial k_m}\delta^4(k)\right] \\ &\times\delta_{dd'}f_{a'cd}\left[(2q+k)^\rho g^{\mu'\lambda} + (-q-2k)^{\mu'}g^{\rho\lambda}\right. \\ &\left.+(k-q)^\lambda g^{\rho\mu'}\right] D^{cc'}_{\lambda\lambda'}(q+k) \times f_{c'b'd'}\left[(2q+k)^\sigma g^{\lambda'\nu'}\right. \\ &\left.+(-q+k)^{\lambda'}g^{\nu'\sigma} + (-2k-q)^{\nu'}g^{\lambda'\sigma}\right] D^{b'b}_{\nu'\nu}(q). \end{aligned} \quad (37)$$

According to the relation between the effective interaction operator and scattering operator

$$V = iS \quad (38)$$

Equation (37) can be rewritten as a relation between the corresponding quark potential:

$$V_{1({\rm a})}(q) = \frac{N_c 4\pi\alpha_s}{N_c^2-1}\left(\frac{\langle 0|B^2|0|\rangle}{q^2} + \frac{192S}{5q^4}\right)V_{qq}^{\rm OGEP}(q), \quad (39)$$

where $V_{qq}^{\rm OGEP}(q)$ is the usual perturbative quark-quark potential arising from the one-gluon exchange mechanism. When using the center-of-mass frame in which

$$\begin{aligned} &\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}, \mathbf{p}_1{}' = -\mathbf{p}_2{}', \\ &E_1' = E_1, E_2' = E_2, \\ &\mathbf{q} = \mathbf{p}_1{}' - \mathbf{p}_1 = -(\mathbf{p}_2{}' - \mathbf{p}_2), q^0 = 0, \end{aligned} \quad (40)$$

one can easily obtain

$$V_{qq}^{\rm OGEP}(q) = \frac{\lambda_1^a\lambda_2^a}{4}4\pi\alpha_s\Bigg\{\frac{1}{|\mathbf{q}|^2} - \frac{(m_1+m_2)^2}{8m_1^2m_2^2} + \frac{|\mathbf{p}|^2}{m_1m_2|\mathbf{q}|^2} - \frac{1}{4m_1m_2|\mathbf{q}|^2} \times \left[|\mathbf{q}|^2\sigma_1\cdot\sigma_2 - (\mathbf{q}\cdot\sigma_1)(\mathbf{q}\cdot\sigma_2)\right] + \frac{i}{4m_1m_2|\mathbf{q}|^2}\left[(2+\frac{m_2}{m_1})\sigma_1 + (2+\frac{m_1}{m_2})\sigma_2\right] \cdot(\mathbf{q}\times\mathbf{p})\Bigg\} \quad (41)$$

which can be transformed into an expression in the coordinate representation

$$U_{qq}^{\rm OGEP}(x) = \int\frac{d^4q}{(2\pi)^4}\exp(-iq\cdot x)V_{qq}^{\rm OGEP}(q) = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\alpha_s\Bigg\{\frac{1}{|\mathbf{x}|} - \frac{\pi}{m_1m_2}\left(\frac{(m_1+m_2)^2}{2m_1m_2} + \frac{2}{3}\sigma_1\cdot\sigma_2\right)\delta(\mathbf{x}) + \frac{|\mathbf{p}|^2}{m_1m_2|\mathbf{x}|} - \frac{1}{4m_1m_2|\mathbf{x}|^3} \times \left[\frac{3}{|\mathbf{x}|^2}(\sigma_1\cdot\mathbf{x})(\sigma_2\cdot\mathbf{x}) - (\sigma_1\cdot\sigma_2)\right] - \frac{1}{4m_1m_2|\mathbf{x}|^3}\left[(2+\frac{m_2}{m_1})\sigma_1 + (2+\frac{m_1}{m_2})\sigma_2\right] \cdot(\mathbf{x}\times\mathbf{p})\Bigg\}, \quad (42)$$

where $\delta(t)$ obviously indicates that the potential is an effective one describing the instantaneous interaction. Performing Fourier transformation to $V_{1(\rm a)}(q)$ of (39), we obtain the contribution of the Feynman diagram 1(a) to the quark-quark potential in the coordinate representation $U_{1(\rm a)}(x)$,

$$U_{1(\rm a)}(x) = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\pi\alpha_s^2 \times \left[A_3|\mathbf{x}|^3 + A_1|\mathbf{x}| + A_{-1}|\mathbf{x}|^{-1}\right], \quad (43)$$

where

$$A_3 = \frac{32N_cS}{5(N_c^2-1)}\left(1+\frac{|\mathbf{p}|^2}{m_1m_2}\right), \quad (44)$$

$$A_1 = \frac{48N_cS}{5(N_c^2-1)}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2 + \frac{8N_cS}{5m_1m_2(N_c^2-1)}(8\sigma_1\cdot\sigma_2 - S_{12}) + \frac{24N_cS}{5m_1m_2(N_c^2-1)}\left[\left(2+\frac{m_2}{m_1}\right)\sigma_1 + \left(2+\frac{m_1}{m_2}\right)\sigma_2\right]\cdot(\mathbf{x}\times\mathbf{p}) + \frac{2N_c\langle 0|B^2|0\rangle}{N_c^2-1}\left(1+\frac{|\mathbf{p}|^2}{m_1m_2}\right), \quad (45)$$

$$A_{-1} = \frac{N_c\langle 0|B^2|0\rangle}{2(N_c^2-1)}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2 + \frac{N_c\langle 0|B^2|0\rangle}{6(N_c^2-1)m_1m_2}S_{12} + \frac{2N_c\langle 0|B^2|0\rangle}{3(N_c^2-1)m_1m_2}\sigma_1\cdot\sigma_2 + \frac{N_c\langle 0|B^2|0\rangle}{2(N_c^2-1)m_1m_2} \times \left[\left(2+\frac{m_2}{m_1}\right)\sigma_1 + \left(2+\frac{m_1}{m_2}\right)\sigma_2\right]\cdot(\mathbf{x}\times\mathbf{p}) \quad (46)$$

with $\mathbf{n} = \mathbf{x}/|\mathbf{x}|$ and $S_{12} = 3(\sigma_1\cdot\mathbf{n})(\sigma_2\cdot\mathbf{n}) - \sigma_1\cdot\sigma_2$.

In the fixed-point gauge of the vacuum gluon field, Fig. 1b makes no contribution [9]. However, in the present case, we use the two-gluon VEV with translational invariance (26) which contains not only the derivative term(second term in (26)), which still contributes nothing, but also the $\langle 0|B^2|0|\rangle$ term(first term in (26)), which yields non-zero contribution of the Feynman diagram 1(b) to the effective potential. Similar to the calculation of Fig. 1a, the contribution of the Feynman diagram 1(b) to the effective potential is

$$U_{1(\rm b)}(x) = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\pi\alpha_s^2[B_1|\mathbf{x}| + B_{-1}|\mathbf{x}|^{-1}] \quad (47)$$

with

$$B_1 = -\frac{3N_c\langle 0|B^2|0\rangle}{(N_c^2-1)}\left(1+\frac{|\mathbf{p}|^2}{m_1m_2}\right), \quad (48)$$

$$B_{-1} = -\frac{N_c\langle 0|B^2|0\rangle}{(N_c^2-1)m_1m_2}\Bigg\{\frac{3(m_1+m_2)^2}{4m_1m_2} + \frac{S_{12}}{4} + \sigma_1\cdot\sigma_2 + \frac{3}{4}\left[\left(2+\frac{m_2}{m_1}\right)\sigma_1 + \left(2+\frac{m_1}{m_2}\right)\sigma_2\right] \cdot(\mathbf{x}\times\mathbf{p})\Bigg\} \quad (49)$$

A nonzero value of B_1 due to retaining the $\langle 0|B^2|0\rangle$ term in (24) and (26) adds a linear term in the effective potential.

By means of the two-quark VEV [14], the contribution of the Feynman diagram 1(c) to the S-matrix can be expressed as

$$S_{1(\rm c)}(p_1,p_2;p_1',p_2') = -ig^4[\bar\psi^-(p_2')\gamma^\nu\frac{\lambda_2^b}{2}\psi^+(p_2)] \times[\bar\psi^-(p_1')\gamma^\mu\frac{\lambda_1^a}{2}\psi^+(p_1)] \times\int d^4k\delta^4(k)\langle 0|\bar q_fq_f|0\rangle\left[\frac{1}{4N_c} + \frac{m_f}{16N_c} \times\gamma^\tau\frac{\partial}{\partial k^\tau}\right]\gamma^\rho\frac{\lambda_f^{a'}}{2}S(q+k)\gamma^\sigma\frac{\lambda_f^{b'}}{2}D_{\mu\rho}^{aa'}(q)D_{\nu\sigma}^{bb'}(q), \quad (50)$$

where we have retained the next-to-leading-order term in the full coefficient of $\langle\bar qq\rangle$ component of the nonperturbative two-quark VEV [14]. There exists a similar expression for Fig. 1d. It is easy to find that the contributions of the

Feynman diagrams 1(c) and 1(d) to the effective potential are the same. Therefore, we can extract the effective potential for the scattering of two quarks of different flavors as shown in Fig. 1c and Fig. 1d in the coordinate representation

$$\begin{aligned} U_{1(\mathrm{c})}(x) &= U_{1(\mathrm{d})}(x) \\ &= \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\pi\alpha_s^2\Big[C_1|\mathbf{x}| + C_{-1}|\mathbf{x}|^{-1} \\ &+\sum_f\left(\tilde{C}_0^{(f)} + \tilde{C}_{-1}^{(f)}|\mathbf{x}|^{-1}\right)\mathrm{e}^{-m_f|\mathbf{x}|}\Big], \end{aligned} \quad (51)$$

where

$$C_1 = \left(1+\frac{|\mathbf{p}|^2}{m_1m_2}\right)\sum_f\frac{\langle 0|\bar{q}_f q_f|0\rangle}{N_c m_f}, \quad (52)$$

$$\begin{aligned} C_{-1} &= \frac{1}{4N_c m_1 m_2}\sum_f\frac{\langle 0|\bar{q}_f q_f|0\rangle}{m_f} \\ &\times\Bigg\{\frac{(m_1+m_2)^2}{m_1m_2} + \frac{S_{12}}{3} + \frac{4}{3}\sigma_1\cdot\sigma_2 \\ &+\left[\left(2+\frac{m_2}{m_1}\right)\sigma_1 + \left(2+\frac{m_1}{m_2}\right)\sigma_2\right]\cdot(\mathbf{x}\times\mathbf{p})\Bigg\} \end{aligned} \quad (53)$$

$$\begin{aligned} \tilde{C}_0^{(f)} &= \frac{2}{N_c}\frac{\langle 0|\bar{q}_f q_f|0\rangle}{m_f}\Bigg[\frac{1}{2m_f}\left(1+\frac{|\mathbf{p}|^2}{m_1m_2}\right) \\ &+\frac{m_f(m_1+m_2)^2}{16m_1^2m_2^2} - \frac{m_f}{24m_1m_2}S_{12} \\ &+\frac{m_f}{12m_1m_2}\sigma_1\cdot\sigma_2\Bigg] \end{aligned} \quad (54)$$

and

$$\begin{aligned} \tilde{C}_{-1}^{(f)} &= -\frac{2}{N_c}\frac{\langle 0|\bar{q}_f q_f|0\rangle}{m_f}\Bigg\{\frac{(m_1+m_2)^2}{8m_1^2m_2^2} + \frac{S_{12}}{6m_1m_2} \\ &+\frac{1}{6m_1m_2}\sigma_1\cdot\sigma_2 - \frac{3}{24m_1m_2} \\ &\times\left[\left(2+\frac{m_2}{m_1}\right)\sigma_1 + \left(2+\frac{m_1}{m_2}\right)\sigma_2\right] \\ &\cdot(\mathbf{x}\times\mathbf{p})\Bigg\} \end{aligned} \quad (55)$$

The total quark-quark effective potential is finally obtained by summing up the contributions of all the corresponding diagrams including perturbative and nonperturbative ones:

$$U_{qq}(x) = U_{qq}^{\mathrm{OGEP}}(x) + U_{qq}^{\mathrm{NP}}(x) \quad (56)$$

where $U_{qq}^{\mathrm{NP}}(x)$, the nonperturbative correction to the perturbative quark-quark due to the quark and gluon condensate, can be expressed as

$$\begin{aligned} U_{qq}^{\mathrm{NP}}(x) &= U_{1(\mathrm{a})}(x) + U_{1(\mathrm{b})}(x) + U_{1(\mathrm{c})}(x) + U_{1(\mathrm{d})}(x) \\ &= \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\pi\alpha_s^2\Big[A_3|\mathbf{x}|^3 + (A_1+B_1+2C_1)|\mathbf{x}| \\ &+(A_{-1}+B_{-1}+2C_{-1})|\mathbf{x}|^{-1} \\ &+2\sum_f\left(\tilde{C}_0^{(f)} + \tilde{C}_{-1}^{(f)}|\mathbf{x}|^{-1}\right)\mathrm{e}^{-m_f|\mathbf{x}|}\Big]. \end{aligned} \quad (57)$$

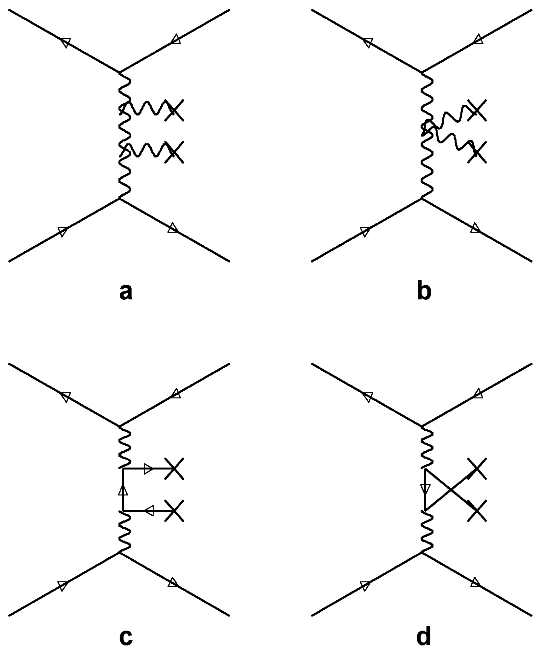

Fig. 2a–d. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative $q\bar{q}$-pair annihilation potential in one-gluon exchange approximation with the lowest dimensional quark and gluon condensates

Formally, (56) holds not only for the qq-, but also for the $q\bar{q}$- and $\bar{q}\bar{q}$-interactions. Note, however, that the color generators for an antiquark are given by $-\lambda^T$, i.e.,

$$U_{q\bar{q}}^{\mathrm{Direct}}(x) = U_{qq}(x)|_{\lambda_1^a\lambda_2^a\to-\lambda_1^a(\lambda_2^a)^T}. \quad (58)$$

and

$$U_{\bar{q}\bar{q}}(x) = U_{qq}(x)|_{\lambda_1^a\lambda_2^a\to(\lambda_1^a)^T(\lambda_2^a)^T}. \quad (59)$$

For an interaction between a quark and an antiquark, if the quark and antiquark are of the same kind of quark fields, then not only the direct scattering but also the annihilation mechanism should be taken into account. The detail discussion is given in the following section.

4 Nonperturbative QCD corrections to the quark-antiquark potential

In the last section, the direct interaction potential between the quark and antiquark was given. When the quark and antiquark have the same kind of flavor, then the annihilation of a quark and an antiquark is possible. The annihilation diagrams including the perturbative and nonperturbative ones shown in Fig. 2 should be further considered. We can obtain the contributions of these Feynman diagrams to the quark-antiquark annihilation potential by means of the same procedure as above. Taking Fig. 2a as an example, the S-matrix for this diagram can be obtained by making the substitutions

$$\bar{\psi}^-(p_2') \to \bar{\psi}^+(p_2), \psi^+(p_2) \to \psi^-(p_2'), m_1 = m_2 = m \quad (60)$$

in (37). In the calculation of color part and spin part in the S-matrix for Fig. 2, it is valuable to note that

$$\sum_{a=1}^{N^2-1} (\lambda^a)_{\beta\alpha}(\lambda^a)_{\alpha'\beta'} = -\frac{1}{N}\sum_{a=1}^{N^2-1} (\lambda^a)_{\alpha'\alpha}(\lambda^a)_{\beta\beta'} + \frac{2(N^2-1)}{N^2}\delta_{\alpha'\alpha}\delta_{\beta\beta'} \quad (61)$$

for $\mathbf{SU}(N)$ generators $\lambda^a(a = 1, 2, \cdots, N^2-1)$, with $N = 3, 2$ for the color and spin generators, respectively. Thus the effective potential for Fig. 2a can be obtained directly as follows,

$$V_{2(\mathrm{a})}(q) = \frac{N_c 4\pi\alpha_s}{N_c^2-1}\left(\frac{\langle 0|B^2|0|\rangle}{(p_1+p_2)^2} + \frac{192S}{5(p_1+p_2)^4}\right) \times V_{q\bar{q}}^{\mathrm{Ann}}(q), \quad (62)$$

where

$$V_{q\bar{q}}^{\mathrm{Ann}}(q) = \frac{4\pi\alpha_s}{(p_1+p_2)^2}\left[\frac{(\lambda_1-\lambda_2^T)^2}{8N_c}\right] \times \left[\frac{(1-\tau_1\cdot\tau_2)}{2}\right]\left\{\frac{(\sigma_1+\sigma_2)^2}{4} \times \left[1-\frac{1}{6m^2}(\mathbf{q}^{\,2}+\mathbf{q}^{\,\prime 2})\right] - \frac{1}{2m^2}\left[(\sigma_1\cdot\mathbf{q})(\sigma_2\cdot\mathbf{q}) + (\sigma_1\cdot\mathbf{q}^{\,\prime})(\sigma_2\cdot\mathbf{q}^{\,\prime}) - \frac{1}{3}\sigma_1\cdot\sigma_2(\mathbf{q}^{\,2}+\mathbf{q}^{\,\prime 2})\right]\right\}. \quad (63)$$

where, $\mathbf{q}$ and $\mathbf{q}^{\,\prime}$ are relative momenta between quarks and antiquarks in the initial and final states respectively. The isospin factor $(1-\tau_1\cdot\tau_2)/2$ in (63) is introduced by considering the fact that the potential has a nonvanishing value only for isospin $T = 0$ state of a quark and antiquark pair, which corresponds to the gluon quantum number. Performing Fourier transformation to $V_{2(\mathrm{a})}(q)$ yields

$$U_{2(\mathrm{a})}(x) = \frac{N_c 4\pi\alpha_s}{N_c^2-1}\left(\frac{\langle 0|B^2|0|\rangle}{4m^2} + \frac{12S}{5m^4}\right)U_{q\bar{q}}^{\mathrm{Ann}}(x), \quad (64)$$

where $U_{q\bar{q}}^{\mathrm{Ann}}(x)$, the perturbative q$\bar{\mathrm{q}}$ pair-annihilation potential in coordinate representation, is,

$$U_{q\bar{q}}^{\mathrm{Ann}}(x) = \delta(t)\frac{\alpha_s}{4}\frac{\pi}{16N_c m^2}(\lambda_1-\lambda_2^{\mathrm{T}})^2(1-\tau_1\cdot\tau_2) \times \left\{(\sigma_1+\sigma_2)^2\left(1-\frac{1}{3m^2}\nabla^2\right)\delta(\mathbf{x}) - \frac{4}{m^2}\left[(\sigma_1\cdot\nabla)(\sigma_2\cdot\nabla) - \frac{1}{3}\sigma_1\cdot\sigma_2\nabla^2\right]\delta(\mathbf{x})\right\}. \quad (65)$$

The nonperturbative contributions of Fig. 2b–d may also be obtained by means of the same procedure as above. The total $q\bar{q}$-pair annihilation potential can be obtained by summing up the contributions of all diagrams including nonperturbative ones in Fig. 2 and the corresponding perturbative one,

$$U_{q\bar{q}}^{\mathrm{Ann(Total)}}(x) = U_{q\bar{q}}^{\mathrm{Ann}}(x) + U_{q\bar{q}}^{\mathrm{Ann(NP)}}(x) \quad (66)$$

where

$$U_{q\bar{q}}^{\mathrm{Ann(NP)}}(x) = \frac{\pi\alpha_s}{m^2}\left\{\frac{N_c}{N_c^2-1}\left[\langle 0|B^2|0\rangle + \frac{48S}{5m^2}\right] + \frac{3N_c\langle 0|B^2|0\rangle}{2(N_c^2-1)} + \frac{1}{N_c}\sum_f \frac{m_f\langle 0|\bar{q}_f q_f|0\rangle}{(4m^2-m_f^2)^2}(8m^2-m_f^2)\right\} \times U_{q\bar{q}}^{\mathrm{Ann}}(x). \quad (67)$$

Therefore, the effective potential between a quark and an antiquark of the same flavor may be expressed as the summation of the direct and annihilation potentials:

$$U_{q\bar{q}\ \text{of the same flavor}}(x) = U_{q\bar{q}}^{\mathrm{Direct}}(x) + U_{q\bar{q}}^{\mathrm{Ann(Total)}}(x) \quad (68)$$

5 Nonperturbative QCD corrections to the q$\bar{\text{q}}$ pair-excitation potential

Now, we turn to the nonperturbative QCD corrections to $q\bar{q}$ pair- excitation potential. By means of the same procedure as the above sections, we can calculate the contribution of Fig. 3 to the effective potential. The result for Fig. 3a reads

$$V_{3(\mathrm{a})}(q) = \frac{N_c}{(N_c^2-1)}4\pi\alpha_s\left[\frac{\langle 0|B^2|0\rangle}{q^2} + \frac{192S}{5q^4}\right] \times V^{q\to qq\bar{q}}(q), \quad (69)$$

where $V^{q\to qq\bar{q}}(q)$, the usual perturbative quark excitation potential in the one-gluon exchange approximation, is [19]

$$V^{q\to qq\bar{q}}(q) = \frac{\lambda_1^a\lambda_2^a}{4}4\pi\alpha_s\frac{1}{q^2}\left[\frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\mathbf{q}\cdot\sigma_2 - \frac{i}{2m_1}\mathbf{q}\cdot(\sigma_1\times\sigma_2) + \frac{\mathbf{p}_1\cdot\sigma_2}{m_1}\right] \quad (70)$$

with $\mathbf{q} = \mathbf{p}_1{}' - \mathbf{p}_1$. As suggested in [19], we adopt two different approximations, i.e., for $q^2 = \omega_q^2 - \mathbf{q}^{\,2}$, $\omega_q = 0$ (case A), and $\omega_q = 2m_2$ with $\mathbf{q} \simeq 0$ (case B). From the Fourier transformation, the expression for the transition potential (70) in the coordinate representation can be written as

$$U^{(\mathrm{A})\mathrm{q}\to\mathrm{qq}\bar{\mathrm{q}}}(x) = -\delta(t)i\alpha_s\frac{\lambda_1^a\lambda_2^a}{4}\frac{1}{2|\mathbf{x}|}\left\{\left[\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\sigma_2 - \frac{i(\sigma_1\times\sigma_2)}{m_1}\right]\cdot\frac{\mathbf{x}}{|\mathbf{x}|^2} - \frac{2i\sigma_2\cdot\mathbf{p}_1}{m_1}\right\}, \quad (71)$$

and

$$U^{(\mathrm{B})\mathrm{q}\to\mathrm{qq\bar{q}}}(x) = -\frac{i\delta(t)}{2m_2^2}\frac{\lambda_1^a\lambda_2^a}{4}\pi\alpha_s\left\{\nabla_{\mathbf{x}}\cdot\left[\frac{\sigma_2}{m_1}+\frac{\sigma_2}{m_2} - \frac{i(\sigma_1\times\sigma_2)}{m_1}\right]\delta(\mathbf{x}) + \frac{2i\sigma_2\cdot\mathbf{p}_1}{m_1}\delta(\mathbf{x})\right\}, \quad (72)$$

in case A and case B, respectively. In case A, the expression for (69) in the coordinate representation is

$$U_{3(\mathrm{a})}^{(\mathrm{A})}(x) = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}4\pi\alpha_s^2[D_3|\mathbf{x}|^3 + D_2|\mathbf{x}|^2 + D_1|\mathbf{x}| + D_0], \quad (73)$$

with

$$D_3 = -\frac{8N_cS}{5(N_c^2-1)m_1}\sigma_2\cdot\mathbf{p}_1, \quad (74)$$

$$D_2 = \frac{12N_cS}{5(N_c^2-1)}\left[\frac{\mathbf{n}\cdot(\sigma_1\times\sigma_2)}{m_1} + i(\frac{1}{m_1}+\frac{1}{m_2})(\sigma_2\cdot\mathbf{n})\right], \quad (75)$$

$$D_1 = -\frac{N_c\langle 0|B^2|0\rangle}{2(N_c^2-1)m_1}\sigma_2\cdot\mathbf{p}_1, \quad (76)$$

and

$$D_0 = \frac{N_c\langle 0|B^2|0\rangle}{4(N_c^2-1)}\left[\frac{\mathbf{n}\cdot(\sigma_1\times\sigma_2)}{m_1} + i(\frac{1}{m_1}+\frac{1}{m_2})(\sigma_2\cdot\mathbf{n})\right]. \quad (77)$$

In case B, one can easily obtain

$$U_{3(\mathrm{a})}^{(B)}(x) = \frac{N_c\pi\alpha_s}{m_2^2(N_c^2-1)}\left(\langle 0|B^2|0\rangle + \frac{48S}{5m_2^2}\right) \times U^{(\mathrm{B})\mathrm{q}\to\mathrm{qq\bar{q}}}(x). \quad (78)$$

Similarly, the potential for Fig. 3b can also be expressed in two cases as follows,

$$U_{3(\mathrm{b})}^{(\mathrm{A})}(x) = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}4\pi\alpha_s^2[E_1|\mathbf{x}| + E_0], \quad (79)$$

in case A, with

$$E_1 = -\frac{3N_c\langle 0|B^2|0\rangle}{4(N_c^2-1)}\frac{\sigma_2\cdot\mathbf{p}_1}{m_1}, \quad (80)$$

and

$$E_0 = \frac{3N_c\langle 0|B^2|0\rangle}{4(N_c^2-1)}\left[\frac{\mathbf{n}\cdot(\sigma_1\times\sigma_2)}{2m_1} + \frac{i}{2}(\frac{1}{m_1}+\frac{1}{m_2})(\sigma_1\cdot\mathbf{n})\right]. \quad (81)$$

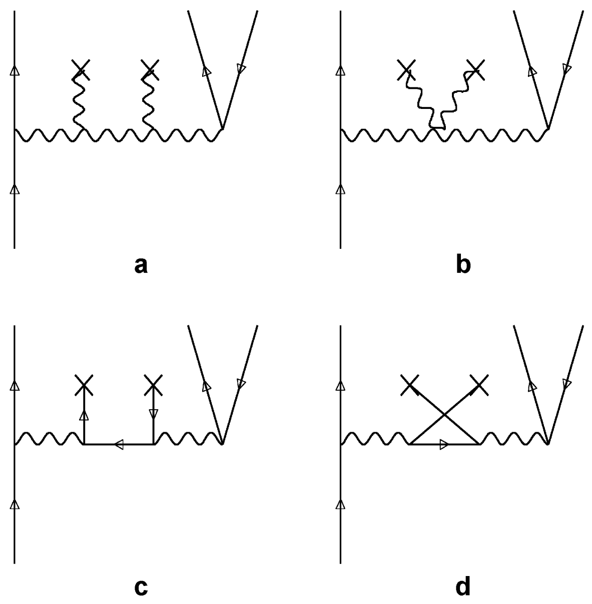

Fig. 3a–d. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative $q\bar{q}$-pair excitation potential in one-gluon exchange approximation with the lowest dimensional quark and gluon condensates

In case B,

$$U_{3(\mathrm{b})}^{(\mathrm{B})}(x) = \frac{3N_c\langle 0|B^2|0\rangle}{2(N_c^2-1)m_2^2}\pi\alpha_s V^{(\mathrm{B})\mathrm{q}\to\mathrm{qq\bar{q}}}(x). \quad (82)$$

The potential for Fig. 3c and that for Fig. 3d are the same, and turn out to be

$$U_{3(\mathrm{c})}^{(A)}(x) = U_{3(\mathrm{d})}^{(A)}(x) = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}4\pi\alpha_s^2 \times\left[F_1|\mathbf{x}| + F_0 + \sum_f \tilde{F}_0^{(f)}\mathrm{e}^{-m_f|\mathbf{x}|}\right], \quad (83)$$

in case A, with

$$F_1 = \frac{\mathbf{p}_1\cdot\sigma_2}{4N_cm_1}\sum_f\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f}, \quad (84)$$

$$F_0 = \frac{i}{8N_c}\sum_f\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f}\left[\frac{i}{m_1}\mathbf{n}\cdot(\sigma_1\times\sigma_2) - \left(\frac{1}{m_1}+\frac{1}{m_2}\right)(\mathbf{n}\cdot\sigma_2)\right], \quad (85)$$

and

$$\tilde{F}_0^{(f)} = \frac{\mathbf{p}_1\cdot\sigma_2}{8N_cm_1}\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f^2} - \frac{i}{16N_c}\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f}\left[\frac{i}{m_1}\mathbf{n}\cdot(\sigma_1\times\sigma_2) - (\frac{1}{m_1}+\frac{1}{m_2})(\mathbf{n}\cdot\sigma_2)\right]. \quad (86)$$

In case B,

$$U_{3(c)}^{(B)}(x) = U_{3(d)}^{(B)}(x) = \frac{\pi\alpha_s}{2N_c m_2^2}\sum_f \frac{m_f\langle 0|\bar{q}_f q_f|0\rangle}{(4m_2^2 - m_f^2)} \times \left[1 + \frac{m_f^2}{2(4m_2^2 - m_f^2)}\right] U^{(B)q\to qq\bar{q}}(x). \quad (87)$$

Therefore, the total transition potential in case A and case B are

$$U_{\mathrm{Total}}^{(A/B)q\to qq\bar{q}}(x) = U^{(A/B)q\to qq\bar{q}}(x) + U^{(A/B)q\to qq\bar{q}(NP)}(x), \quad (88)$$

$U^{(A/B)q\to qq\bar{q}(NP)}(x)$, the nonperturbative corrections to the perturbative q$\bar{q}$-pair excitation potential from all diagrams shown in Fig. 3 are

$$U^{(A)q\to qq\bar{q}(NP)}(x) = U_{3(a)}^{(A)} + U_{3(b)}^{(A)} + U_{3(c)}^{(A)} + U_{3(d)}^{(A)} = \delta(t)\frac{\lambda_1^a\lambda_2^a}{4}4\pi\alpha_s^2\Bigg[D_3|\mathbf{x}|^3 + D_2|\mathbf{x}|^2 + (D_1 + E_1 + 2F_1)|\mathbf{x}| + (D_0 + E_0 + 2F_0) + 2\sum_f \tilde{F}_0^{(f)}\mathrm{e}^{-m_f|\mathbf{x}|}\Bigg], \quad (89)$$

in case A, and

$$U^{(B)q\to qq\bar{q}(NP)}(x) = U_{3(a)}^{(B)} + U_{3(b)}^{(B)} + U_{3(c)}^{(B)} + U_{3(d)}^{(B)} = \frac{\pi\alpha_s}{m_2^2}\Bigg\{\frac{N_c}{(N_c^2-1)}\left(\langle 0|B^2|0\rangle + \frac{48S}{5m_2^2}\right) + \frac{3N_c\langle 0|B^2|0\rangle}{2(N_c^2-1)} + \frac{1}{N_c}\sum_f \frac{m_f\langle 0|\bar{q}_f q_f|0\rangle}{(4m_2^2 - m_f^2)}\left[1 + \frac{m_f^2}{2(4m_2^2 - m_f^2)}\right]\Bigg\} \times U^{(B)q\to qq\bar{q}}(x). \quad (90)$$

in case B.

6 Discussion and summary

The non-local two-gluon VEV is essential in the nonperturbative calculation. However, this VEV in the fixed-point gauge violates the translational invariance. We gave in Sect. 2 the non-local two-gluon VEV with translational invariance in Lorentz gauge. In order to include the nonperturbative corrections in a more self-consistent way in the nonrelativistic potential reduction, By using the obtained two-gluon VEV, the quark-quark, $q\bar{q}$ pair-annihilation and excitation-type potentials are presented with nonperturbative corrections.

Furthermore, some new features of the resultant quark-quark potential appear: A linear term which can play a role of the regular confinement potential comes from both quark condensate and the gluon condensate corrections to the gluon propagators; a cubic term $|\mathbf{x}|^3$ results only from the nonperturbative gluon propagator with the gluon condensate modification; A Yukawa-type term $|\mathbf{x}|^{-1}\mathrm{e}^{-m_f|\mathbf{x}|}$ which may somewhat provide an interaction at longer range arises form the nonzero quark condensate; a Coulomb term $|\mathbf{x}|^{-1}$ comes from the nonvanishing $\langle 0|B^2|0\rangle$ and quark condensate. In the $q\bar{q}$ excitation potential, the linear, square and cubic terms also appear in the case of $\omega_q = 0$ due to the nonzero quark and gluon condensates.

It is noteworthy that the framework of the nonperturbative calculation is an extrapolatation from short distances where perturbative QCD is reliable. Therefore, the nonperturbative correction can not be expected to include as much as a purly phenomenological ansatz. But, it does enrich our understanding of hadronic structure and shed light on the underlying mechanism which determines how quarks are bound into hadrons.

As an extension of this work, we will verify whether the potentials obtained here can be used to improve the hadronic spectra and hadronic properties of J/Ψ and Υ families. Furthermore, the nonperturbative corrections are comparable with those from perturbative closed-loop as shown by Gupta et al. [21], Fulcher [22] and Pantaleone et al. [23] since these two kinds of corrections are in the same order of α_s. Therefore, in a complete analysis, not only perturbative closed-loop contributions, but also nonperturbative corrections presented in this paper should be taken into account.

Physically relevant results, such as the effective quark-quark interaction potential, should be gauge-independent. The difference between the present result of quark-quark interaction and that of [9] indicates that there are still gauge issues to be clarified. For this reason, we should fulfill the construction of description in Lorentz gauge by including the ghost propagators and ghost condensates as well as the $\langle B^2\rangle$ condensates according to the Slavnov-Taylor identities (STI) [24]. We hope that the nonperturbative correction to the quark potential can be obtained with the translational invariance as well as gauge-independence. Further studies along this direction are in progress.

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